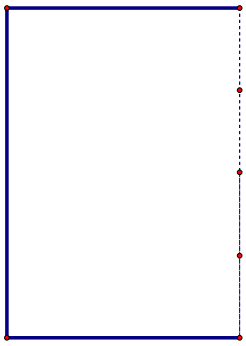
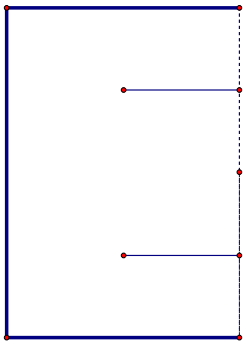
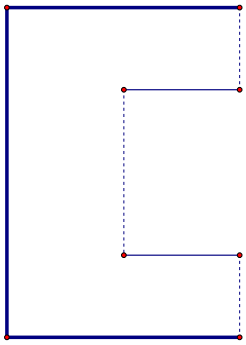
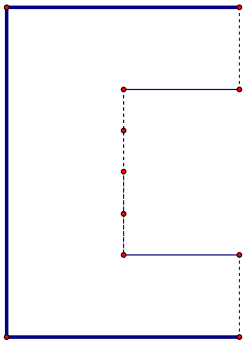
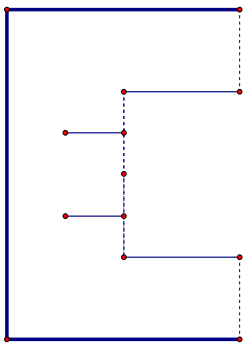
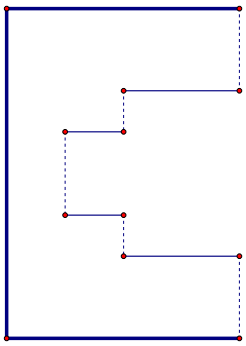
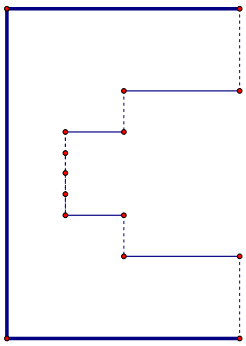
Resource sheet 1.1 **Fractals**

	Hold a piece of A4 paper in the landscape orientation and then fold in half.		Measure accurately along the folded edge, the midpoint and then the quarter points.
	From the top and bottom quarter measurements measure accurately half way back. Make sure that your lines are parallel to the top of the paper. Cut along the two lines.		Fold the cut flap of paper 'inside' so that a C shape remains.
	On the new folded edge measure accurately the midpoint and then the quarter points.		From the top and bottom quarter measurements measure accurately half way back. Make sure that your lines are parallel to the top of the paper. Cut along the two lines.
	Fold each of the 2 cut sections back inside the fractal.		On the new folded edge measure accurately the midpoint and then the quarter points.

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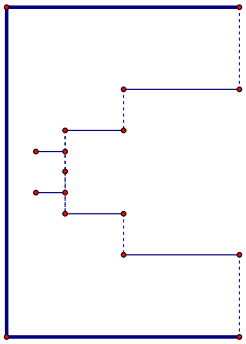
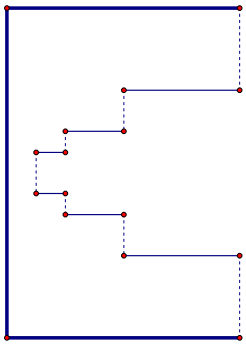
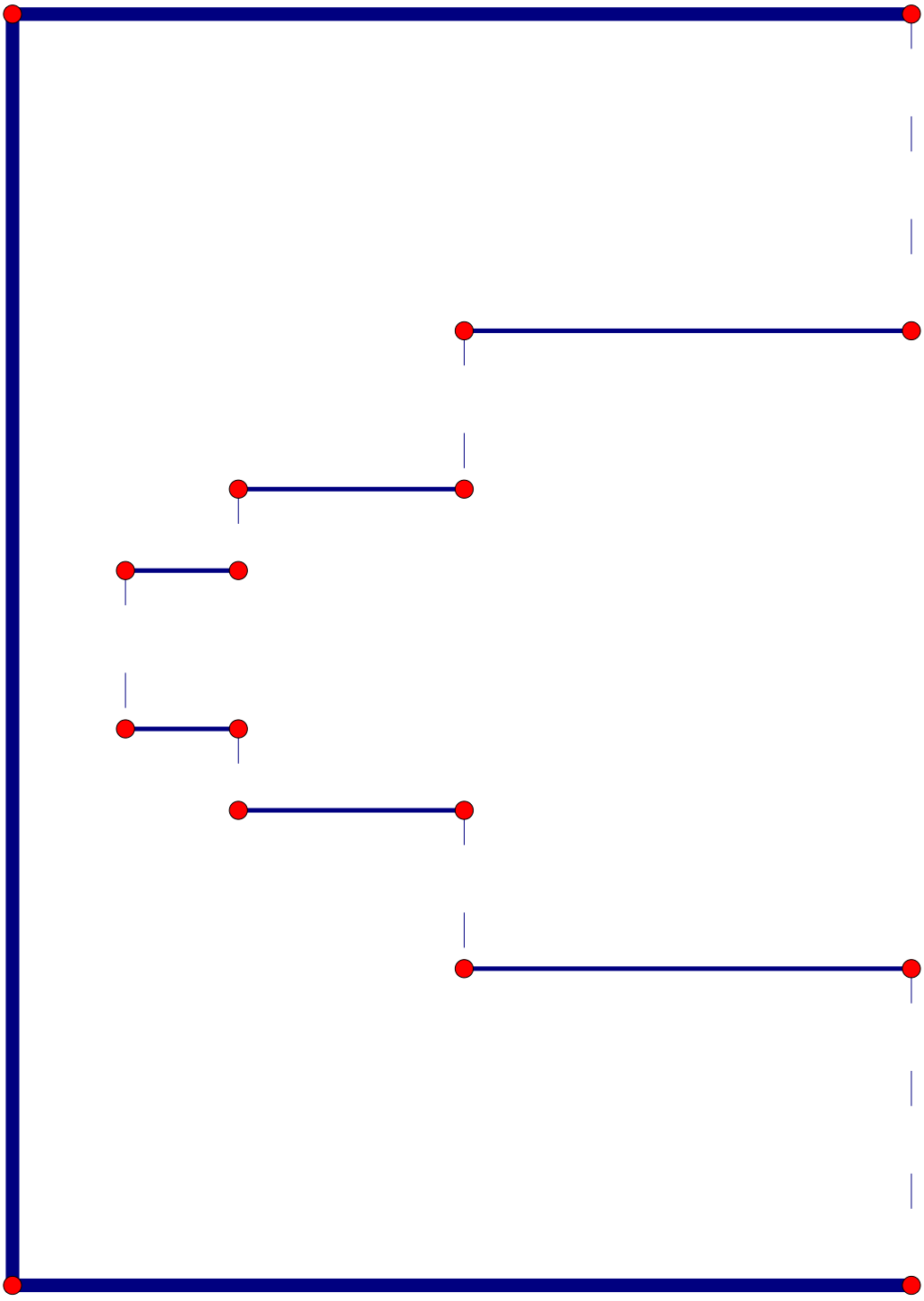
 From the top and bottom quarter measurements measure accurately half way back. Make sure that your lines are parallel to the top of the paper. Cut along the two lines.	 Fold each of the 4 cut sections back inside the fractal. Your 3D fractal is now complete. Open the paper to view the result.
Take a piece of A4 paper and fold it in half, by placing the shorter edges together.	Measure accurately along the folded edge and mark the midpoint and the quarter-points.
From the top and bottom quarter-points, measure accurately halfway towards the edges, making sure that your lines are parallel to the top of the paper. Cut along the two lines.	Fold the cut flap of paper 'inside' so that a C shape remains.
On the new folded edge, measure accurately the midpoint and then the quarter-points.	From the top and bottom quarter-points, measure accurately half way back. Make sure that your lines are parallel to the top of the paper. Cut along the two lines.
Fold each of the two cut sections back inside the fractal.	On the new folded edge, measure accurately the midpoint and then the quarter-points.
From the top and bottom quarter-points, measure accurately halfway back. Make sure that your lines are parallel to the top of the paper. Cut along the two lines.	Fold each of the four cut sections back inside the fractal. Your 3D fractal is now complete. Open the paper to view the result.

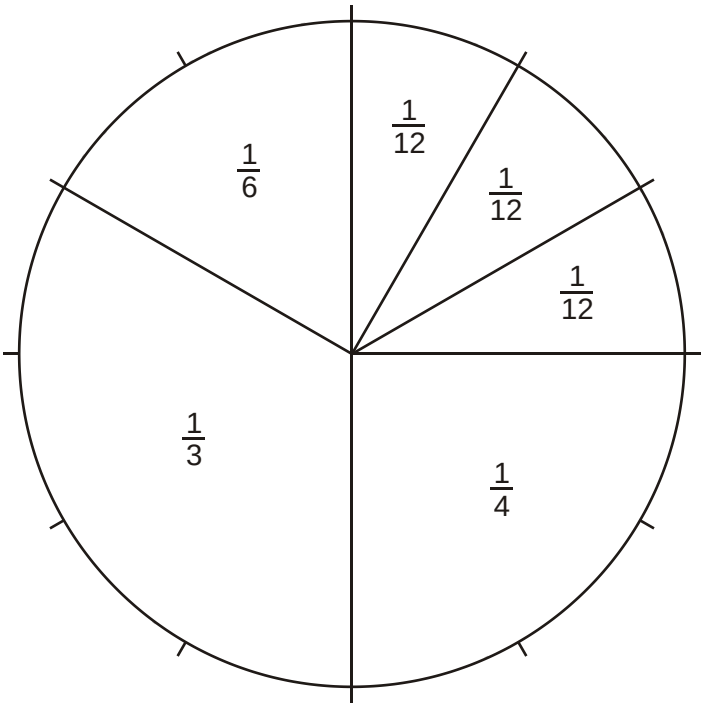
Resource sheet 1.2 **Fractals**



Resource sheet 1.3 **Fractals**

1. Fractions

Look at this diagram.



The diagram can help you work out some fraction calculations.

Calculate:

$\frac{1}{12} + \frac{1}{4} =$

1 mark

$\frac{1}{3} + \frac{1}{4} =$

1 mark

$\frac{1}{3} - \frac{1}{6} =$

1 mark

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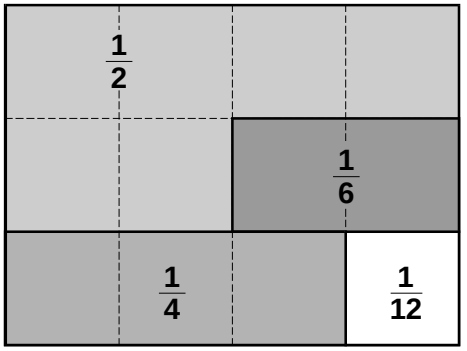
Resource sheet 1.4 **Fractals**

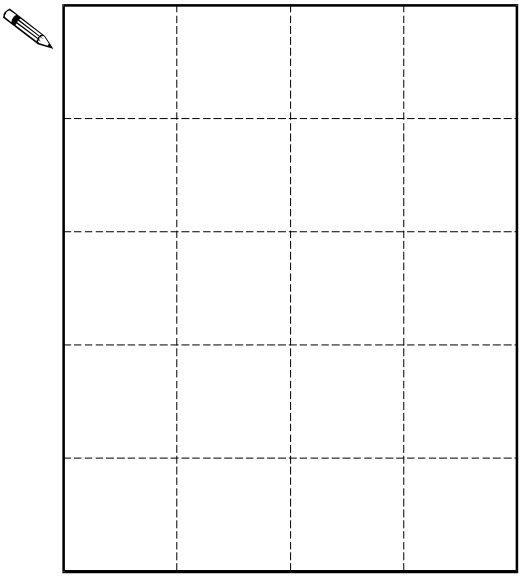
1. Unit fractions

The diagram shows that $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{12}=1$



Draw lines on the rectangle below to show that $\frac{1}{2}+\frac{1}{4}+\frac{1}{5}+\frac{1}{20}=1$

Label each part with its fraction.



2 marks

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Resource sheet 1.5 **Fractals**

1. Adverts

(a) In a magazine, there are three adverts on the same page:

Advert 1 uses $\frac{1}{4}$ of the page

Advert 2 uses $\frac{1}{8}$ of the page

Advert 3 uses $\frac{1}{16}$ of the page

In total, what **fraction** of the page do the three adverts use?
Show your working.

2 marks

(b) Cost of advert: **£10** for each $\frac{1}{32}$ of a page

An advert uses $\frac{\mathbf{3}}{\mathbf{16}}$ of a page. How much does the advert cost?

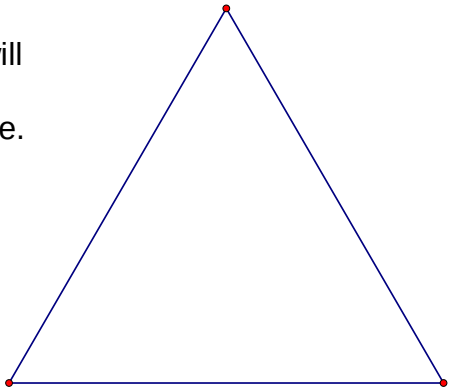
£ .....……..

1 mark

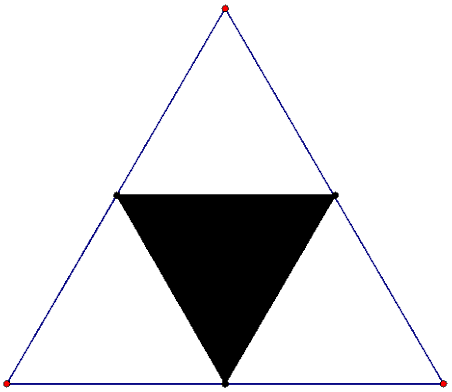
Total: 3 marks

Resource sheet 2.1 **Fractals**

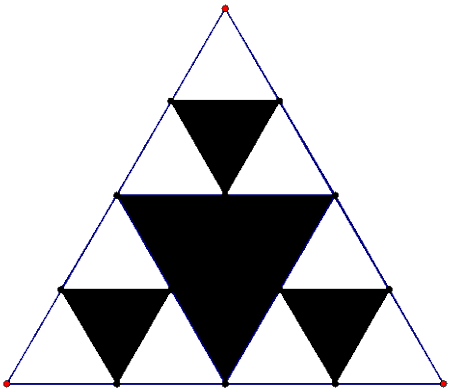
Write an algorithm that will produce this fractal. It is called Sierpinski's triangle.



Step 1

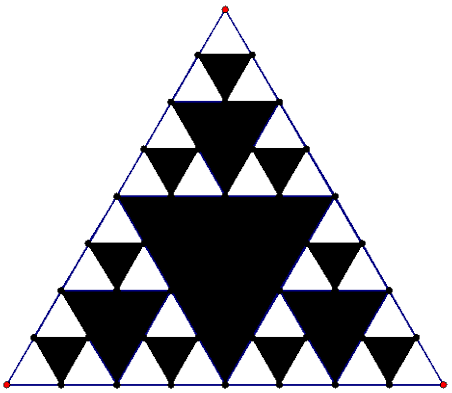


Step 2



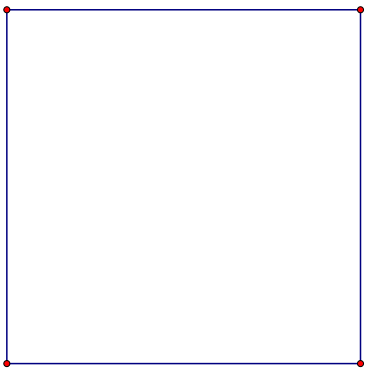
Step 3

Step 4

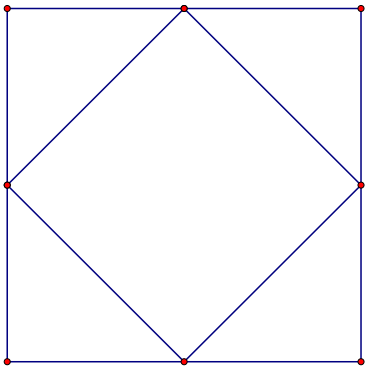


Resource sheet 2.2 **Fractals**

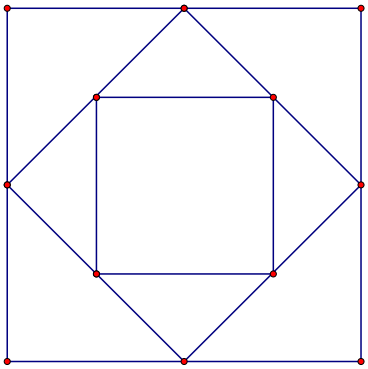
Write an algorithm to produce this fractal.



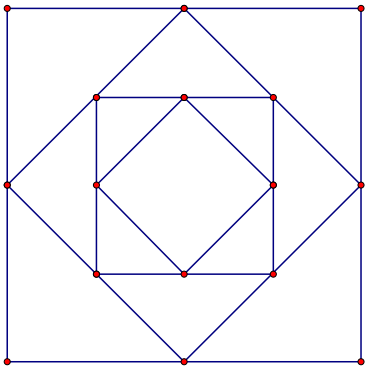
Step 1



Step 2



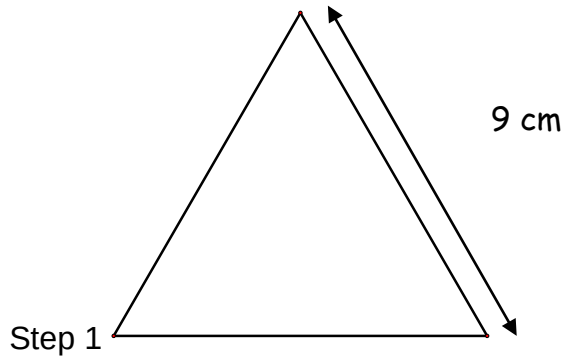
Step 3



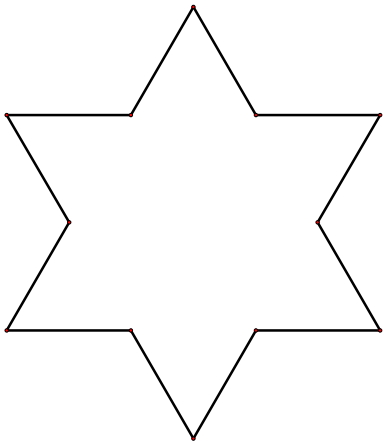
Step 4

Resource sheet 3.1

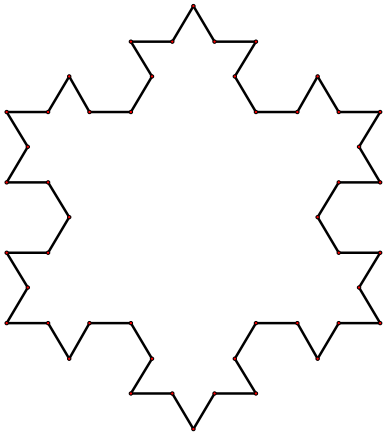
Fractals



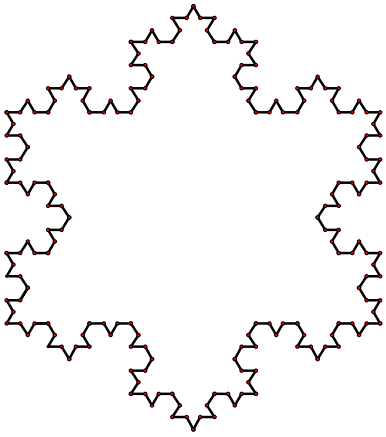
Step 1



Step 2



Step 3



Step 4

Resource sheet 3.2 Fractals

Pattern number	Number of edges	Length of each edge	Total perimeter
1			
2			
3			
4			
5			

Resource sheet 3.3 Fractals

Pattern number	Number of edges	Length of each edge	Total perimeter
1	3	9	27
2	12	3	36
3	48	1	48
4	192	$\frac{1}{3}$	64
5	768	$\frac{1}{9}$	$85\frac{1}{3}$
10			
n			

Resource sheet 3.5 **Fractals**

Value of C5 can be calculated by	D4÷3
	=D4÷3
	3×4
	=C1*4
	=C6/4
	=C4×4
Value of D5 can be calculated by	=C5×D5
	=C4*D4
	=C4*4
	27
	=D6*3
Value of E4 can be calculated by	=D4*4
	C1 x 4
	=D4/3
	C4*4
	C4×4

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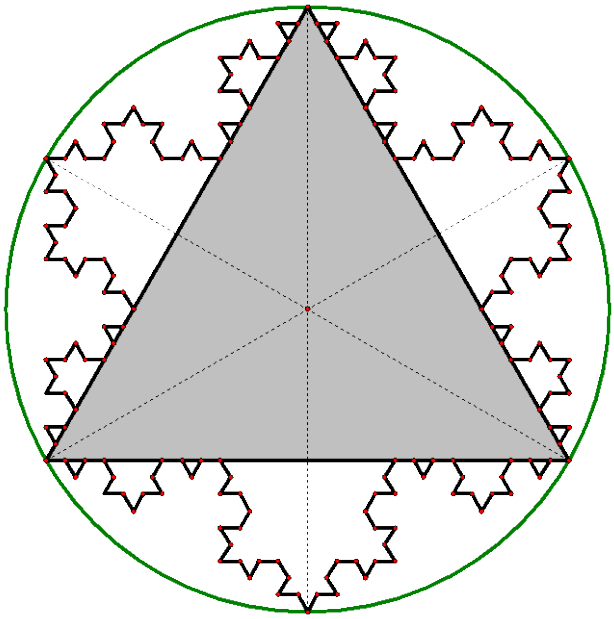
Resource sheet 3.6 Fractals

Use the diagram below of step 4 of the snowflake.

1. Measure the length of the sides and height of the equilateral triangle.
2. Calculate the area of the equilateral triangle in cm^2.
3. Look at the circle centred on the midpoint that encloses the snowflake.

The area of the circle is approximately 50 cm^2.

4. What can you say about the area of the von Koch snowflake?



Area and perimeter

How to use these resources

In a departmental meeting:

- consider the pupil's responses to the questions set (resource C1) and discuss the misconceptions that are evident;
- compare your response with the findings from the researchers (resource C2);
- predict how the pupil might answer an associated test question (resource C3);
- explore approaches that target the misconceptions (resource C4) and do the card sort activity (resource C5);
- discuss likely outcomes from pupils' discussions when they use the card sort;
- consider how to use pupils' responses to create and resolve the cognitive conflict by encouraging them to discuss their imagery and reasoning.

In teaching:

- consolidate skills by setting questions that focus on the application of the newly acquired concept;
- adjust your schemes of work to incorporate the activities designed to counter misconceptions.

Samples of a pupil's work

3. Explain, in your own words, the meanings of the terms:

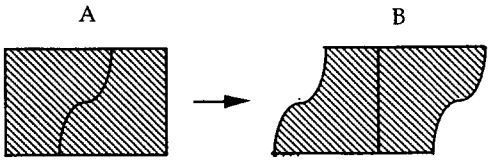
Perimeter Is the outside of a shape

Area The flat surface of a shape

Volume The length + surface area eg. the whole shape.

eg Inside of shape

4. You cut rectangle A, and arrange the pieces to make a new shape B, like this:



Ring two statements that are true:

The area of A is greater than the area of B

The area of A is less than the area of B

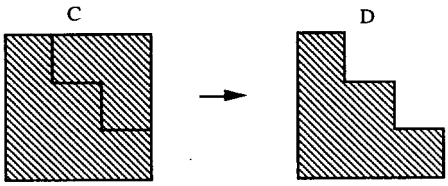
(Both areas are the same)

The perimeter of A is greater than the perimeter of B

The perimeter of A is less than the perimeter of B

(Both perimeters are the same)

5. You cut a piece out of C and throw the piece away.
You are left with piece D:



Ring two statements that are true:

(The area of C is greater than the area of D)

The area of C is less than the area of D

Both areas are the same

(The perimeter of C is greater than the perimeter of D)

The perimeter of C is less than the perimeter of D

Both perimeters are the same

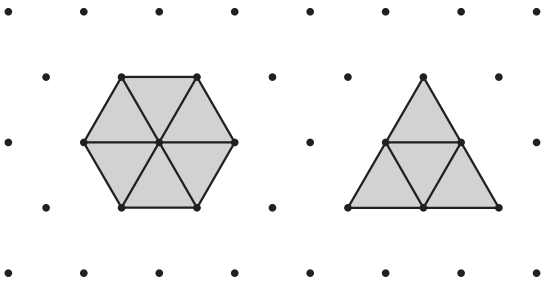
Commentary on the pupil's work

The pupil's answers to question 3 show that she is able to distinguish area from perimeter which, in her words, mean 'the flat surface of a shape' and 'the outside of a shape' respectively. She appears to have difficulty explaining the concept of volume, which she describes as the 'whole shape', including length, surface area and inside the shape. Her drawings indicate that she relates these concepts to the number of dimensions involved.

Her answers to question 4 indicate that she knows that area is conserved when a shape is cut up and reassembled, but she seems to think that perimeter also is conserved. Her response to question 5 suggests she believes that if the area of a shape is increased, then so the perimeter must also increase. She may thus believe that there is a relationship between the area and the perimeter of a shape.

Key Stage 3 test questions

1 Look at the hexagon and the triangle.



(a) Do the hexagon and the triangle have the same area?
Tick (✓) Yes or No.

Yes ☐ No ☐

Explain your answer.

_____

_____

(b) Do the hexagon and the triangle have the same perimeter?
Tick (✓) Yes or No.

Yes ☐ No ☐

Explain your answer.

_____

_____

2 The information in the box describes three different squares, A, B and C.

> The area of square A is 36 cm^2.
>
> The side length of square B is 36 cm.
>
> The perimeter of square C is 36 cm.

Put squares A, B and C in order of size, starting with the smallest.

You must show calculations to show how you worked out your answer.

_____ _____ _____

smallest largest

Always, sometimes or never true?

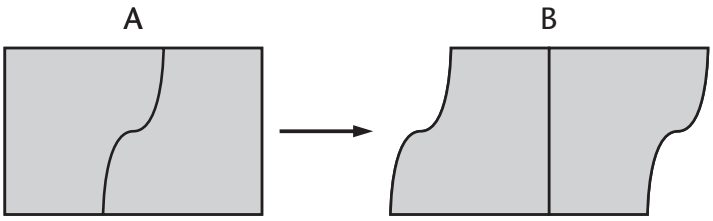
Do these three questions **on your own**, without talking to your neighbour.

1 Explain in your own words what is meant by **perimeter** and **area**.

2 Draw a shape with a big perimeter and a small area.

3 Draw a shape with a small perimeter and a big area.

4 You cut rectangle A and arrange the pieces to make a new shape B, like this.



Tick (✓) two statements that are true:

- ☐ The area of A is greater than the area of B.
- ☐ The area of A is less than the area of B.
- ☐ Both areas are the same.
- ☐ The perimeter of A is greater than the perimeter of B.
- ☐ The perimeter of A is less than the perimeter of B.
- ☐ Both perimeters are the same.

Try the following activity in a small group.
You will need the cards for sorting from resource C5.

5 Divide a sheet of paper into three columns. Label them 'Always true', 'Sometimes true', 'Never true'.

Decide whether each of the statements on the cards is

- Always true (it is true for all possible shapes)
- Sometimes true (it is true for just some shapes)
- Never true (no shapes make the statement true)

Take it in turns in your group to explain your thinking.

When you reach agreement, stick the statement into the correct column.

Explain using examples and drawings how you made your decision. Write these reasons directly on the poster, next to the statement.

On your own, go back and revise your answers to questions 1 to 4.
Make notes on any mistakes you made and the reasons for them.
Make notes on new things you have learned about area and perimeter.

Statements for sorting

C1 Draw two rectangles. The one with the greater area will also have the greater perimeter.
C2 If you cut a piece out of a rectangle, you make its area smaller.
C3 If you cut a piece out of a rectangle, you make its perimeter smaller.
C4 A square and a rectangle both have the same perimeter. The square has the greater area.
C5 A square and a rectangle both have the same area. The square has the greater perimeter.

Resource sheet 4.2 Fractals

3	3×4^0	9	$9 \div 3^0$
3×4	3×4^1	$9 \div 3$	$9 \div 3^1$
$3 \times 4 \times 4$	3×4^2	$(9 \div 3) \div 3$	$9 \div 3^2$
$3 \times 4 \times 4 \times 4$	3×4^3	$((9 \div 3) \div 3) \div 3$	$9 \div 3^3$
$3 \times 4 \times 4 \times 4 \times 4$	3×4^4	$(((9 \div 3) \div 3) \div 3) \div 3$	$9 \div 3^4$

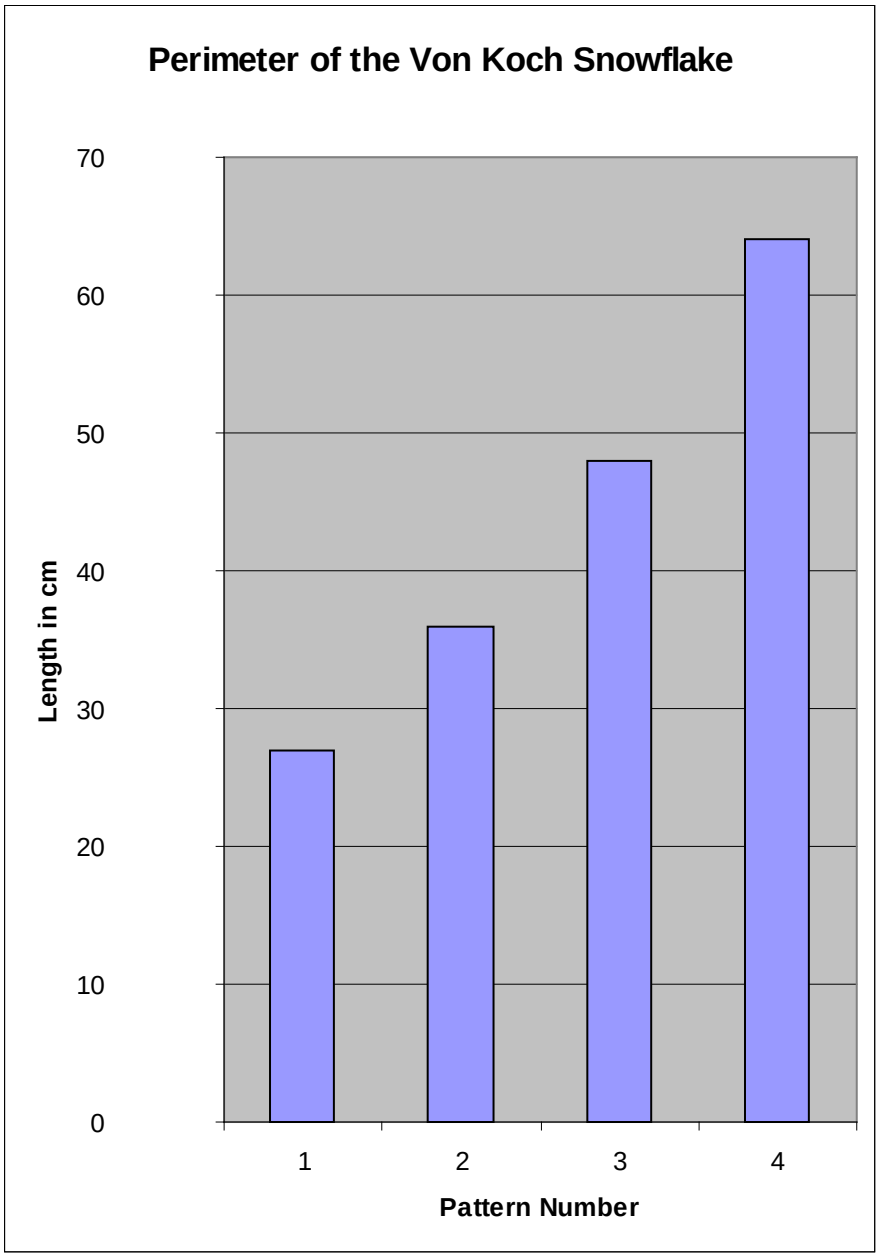
Resource sheet 4.3 Fractals

Pattern number	Number of edges			Length of each edge		
1	3			9		
2	12			3		
3	48			1		
4	192			$\frac{1}{3}$		
5	768			$\frac{1}{9}$		
10						
n						

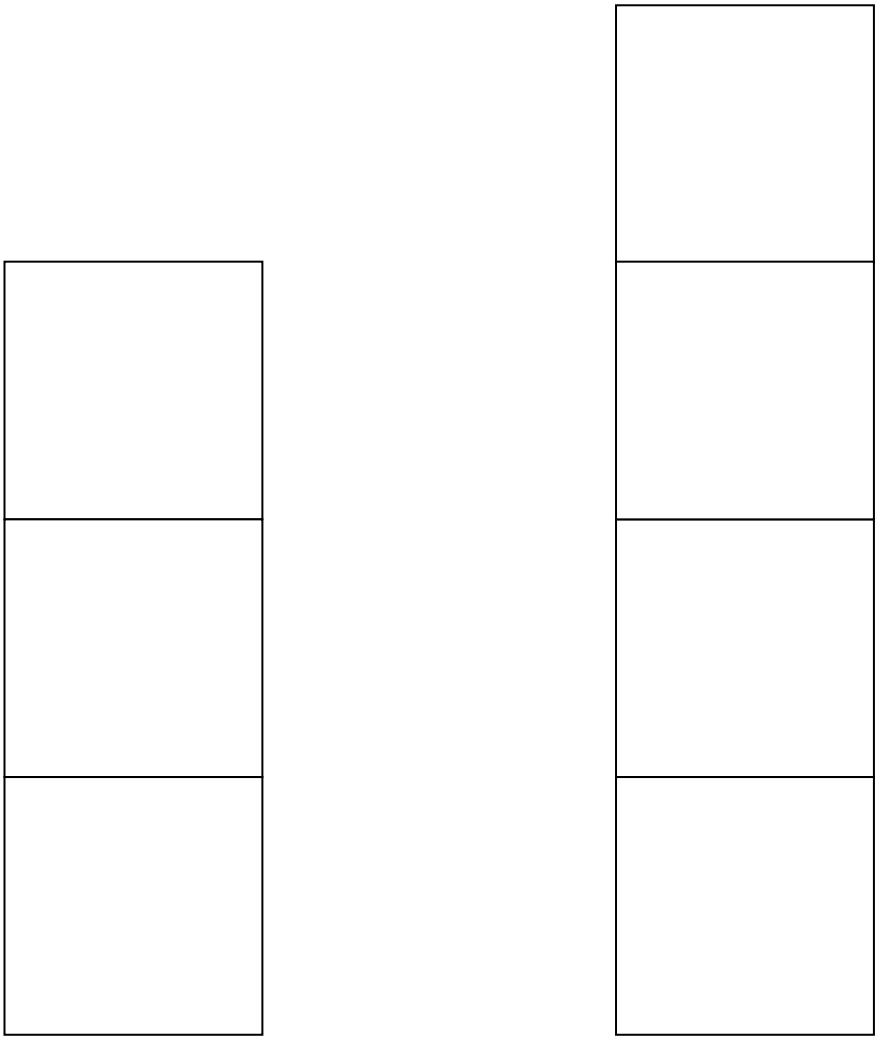
Resource sheet 4.4 Fractals

Pattern number	Number of edges			Length of each edge		
1	3	3	3×4^0	9	9	$9 \div 3^0$
2	12	3×4	3×4^1	3	$9 \div 3$	$9 \div 3^1$
3	48	$3 \times 4 \times 4$	3×4^2	1	$(9 \div 3) \div 3$	$9 \div 3^2$
4	192	$3 \times 4 \times 4 \times 4$	3×4^3	$\frac{1}{3}$	$((9 \div 3) \div 3) \div 3$	$9 \div 3^3$
5	768	$3 \times 4 \times 4 \times 4 \times 4$	3×4^4	$\frac{1}{9}$	$(((9 \div 3) \div 3) \div 3) \div 3$	$9 \div 3^4$
10						
n						

Resource sheet 4.5 Fractals



Resource sheet 4.6 Fractals



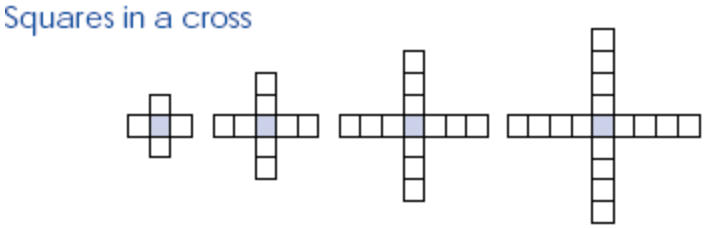
Resource sheet 4.7 Fractals

START CARD Perimeter of pattern 4 is given by:	$number\ of\ edges\ in\ pattern\ 4 \times length\ of\ each\ edge\ in\ pattern\ 4$
$192 \times {}^{1}/_{3}$	I can rewrite this using the expressions that were generated in the last activity.
$(3 \times 4^3) \times (9 \div 3^3)$	I can write this expression without the brackets and it will still give the same answer.
$3 \times 4^3 \times 9 \div 3^3$	I can multiply numbers in any order and still get the same answer.
$3 \times 9 \times 4^3 \div 3^3$	Using my knowledge of multiplication facts I know this is the same as:
$27 \times 4^3 \div 3^3$	Using my knowledge of indices this is the same as:
$27 \times (4 \times 4 \times 4) \div (3 \times 3 \times 3)$	Using the facts that division can be written using fraction notation.
$27 \times \frac{4 \times 4 \times 4}{3 \times 3 \times 3}$	This can be written using separate fractions.
$27 \times \frac{4}{3} \times \frac{4}{3} \times \frac{4}{3}$	Using my knowledge of indices this is the same as:
$27 \times \left(\frac{4}{3}\right)^3$	This is the simplest expression for the perimeter of the 4th pattern that will help me derive the position-to-term rule.

Resource sheet 5.1 **Fractals**

Sequences, functions and graphs

Practical context



Sequence

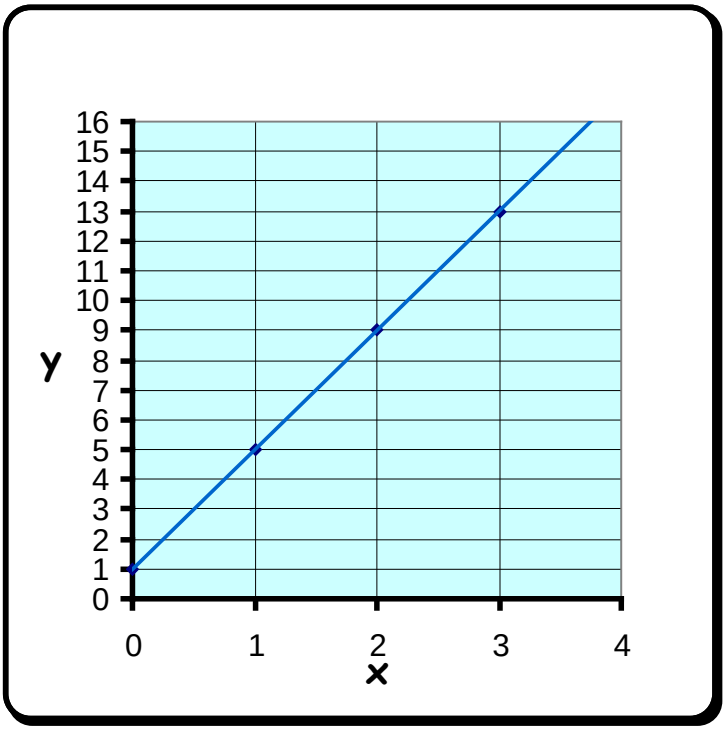
5, 9, 13, 17, …, … *n*th term is $4n + 1$

Function

$y = 4x + 1$

x = pattern number
y = total number of squares

Graph



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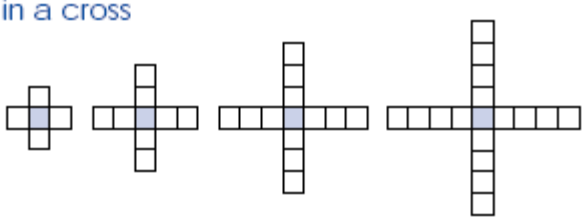
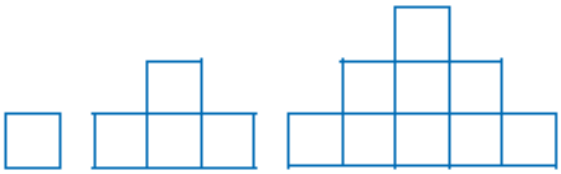
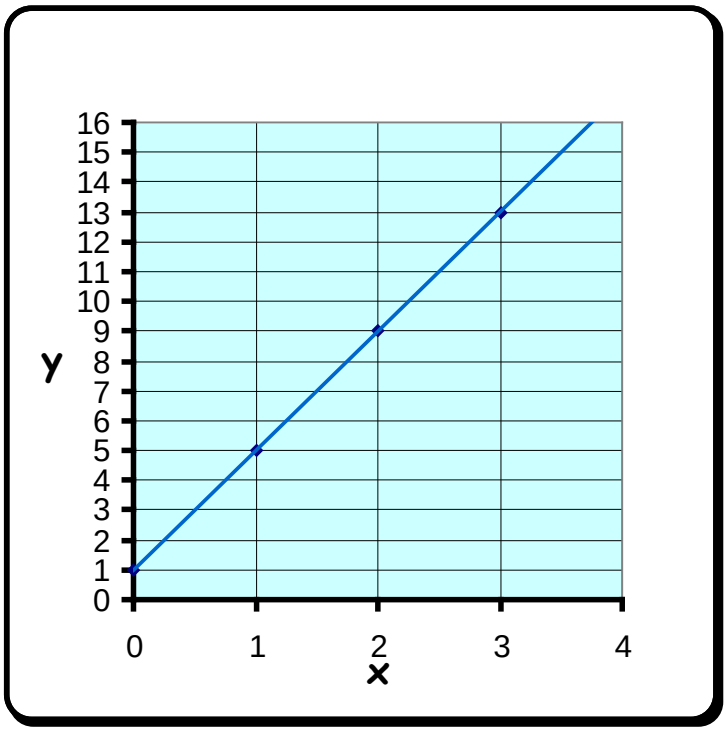
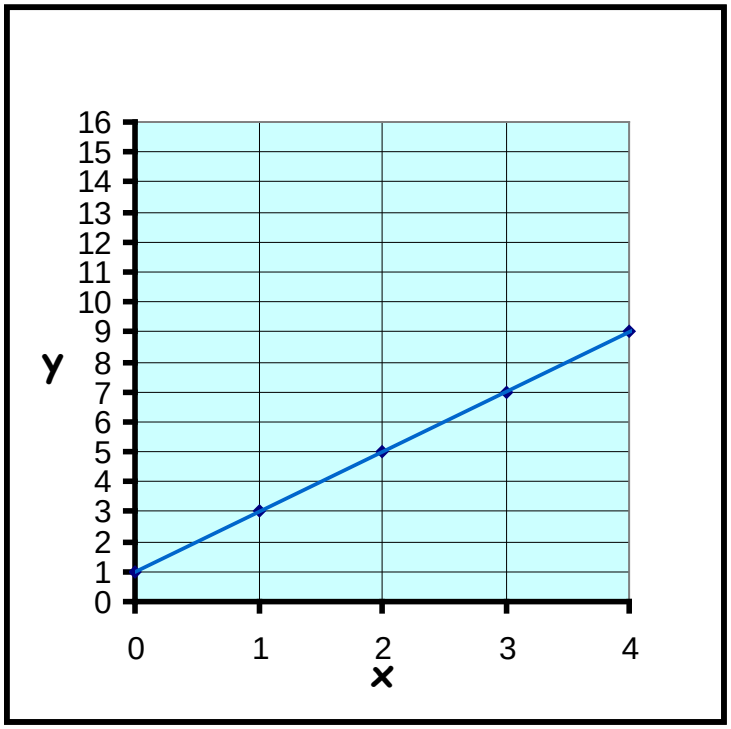
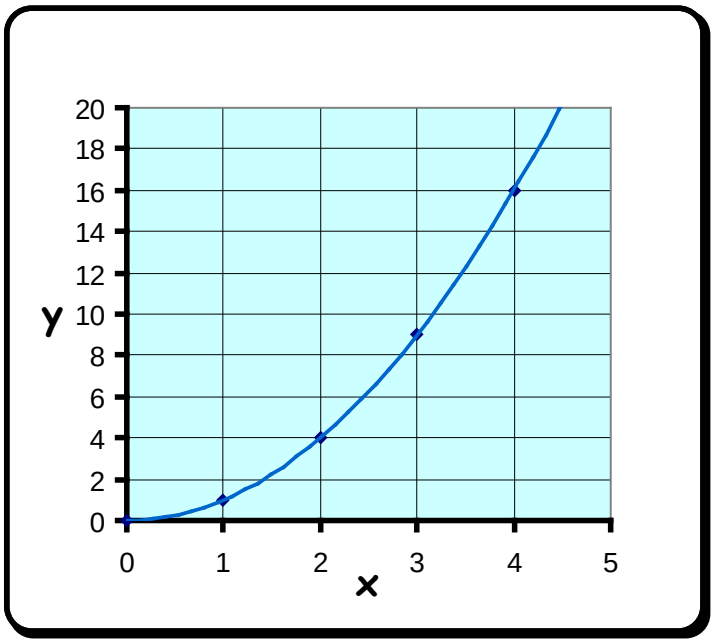
Resource sheet 5.2 **Fractals**

Real-life application	Sequences
Growing matchstick squares 	4, 7, 10, 13, ..., …
Squares in a cross 	3, 12, 48, …, …

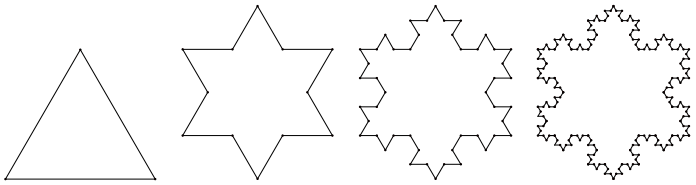
'Pyramid' of squares 	1, 4, 9, 16, …, …
	3, 5, 7, 9, 11, …, …
Graphs	**Functions**



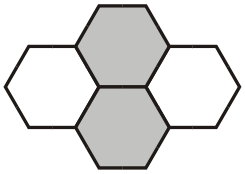
$y = 3x + 1$



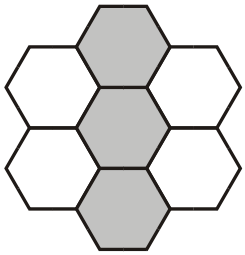
$y = 3 \times 4^{n-1}$

	$y = 4x + 1$
	$y = 2x + 1$

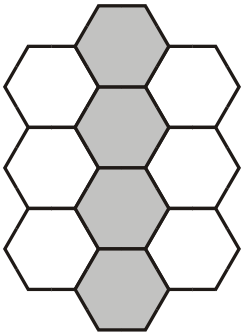
Resource sheet 5.3 Fractals



pattern number 1



pattern number 2



pattern number 3

Resource sheet 5.4 Fractals

Question 1

Here are the first five numbers of a simple sequence:

1, 5, 9, 13, 17, …

Write down the next two numbers of the sequence.

Write down, in terms of the position number (n), an expression for the nth term in this sequence.

Draw a graph and a possible practical context or growing geometrical pattern for this sequence.

Question 2

Here are the first five terms of an arithmetic sequence:

6, 11, 16, 21, 26, …

Write down the term-to-term rule for the sequence.

Find an expression, in terms of n, for the nth term of the sequence.

Draw a graph and a possible practical context or growing geometrical pattern for this sequence.

Resource sheet 6.1 Fractals

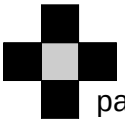
Pupil self-evaluation grid

Objective		Example from the *Fractal* unit of work	Example from a different context
I know how to describe simple sequences			
I can find the term-to-term rule for simple sequences			
I can find the position-to-term rule for simple sequences			
I can			
I can			

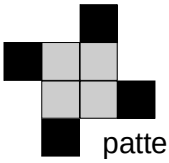
Resource sheet 6.2 **Fractals**

1. Windmills

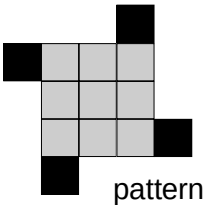
This is a series of patterns with grey and black tiles.



pattern number 1



pattern number 2



pattern number 3

(a) How many grey tiles and black tiles will there be in pattern number 8?

..... **grey** tiles and ..... **black** tiles

1 mark

(b) How many grey tiles and black tiles will there be in pattern number 16?

..... **grey** tiles and ..... **black** tiles

1 mark

(c) How many grey tiles and black tiles will there be in pattern number P?

..... **grey** tiles and ..... **black** tiles

1 mark

(d) T = total number of grey tiles and black tiles in a pattern
P = pattern number

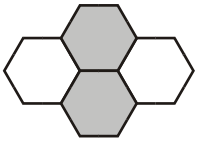
Use symbols to write down an equation connecting T and P.

1 mark

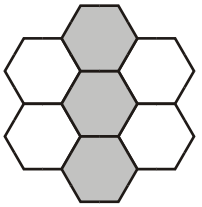
Resource sheet 6.3 **Fractals**

1. Tiles

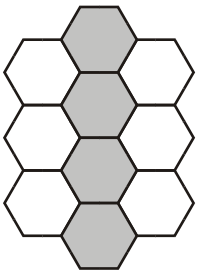
This is a series of patterns with grey and white tiles.



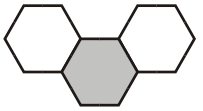
pattern number 1



pattern number 2



pattern number 3

The series of patterns continues by adding  each time.

(a) Complete this table:

pattern number	number of **grey** tiles	number of **white** tiles
5		
16		

2 marks

(b) Complete this table by writing **expressions**:

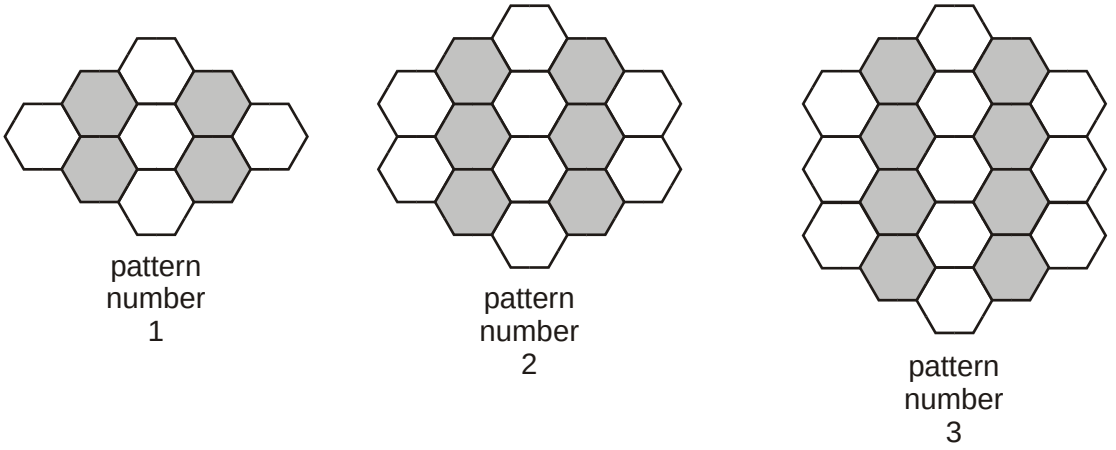
pattern number	expression for the number of grey tiles	expression for the number of **white** tiles
n		

2 marks

(c) Write an expression to show the **total** number of tiles in pattern number n.
Simplify your expression.

1 mark

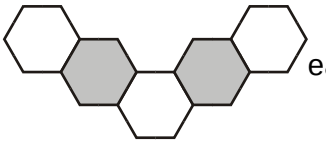
(d) A different series of patterns is made with tiles.



pattern number 1

pattern number 2

pattern number 3

The series of patterns continues by adding  each time.

For this series of patterns, write an expression to show the **total** number of tiles in pattern number n.

Show your working and **simplify** your expression.

2 marks

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Resource sheet 6.4a **Fractals**

1. Sequence

Each term of a number sequence is made by adding 1 to the numerator and 2 to the denominator of the previous term.

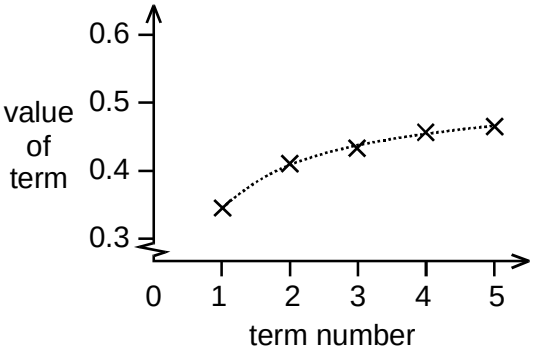
Here is the beginning of the number sequence:

$$\frac{1}{3} \quad , \quad \frac{2}{5} \quad , \quad \frac{3}{7} \quad , \quad \frac{4}{9} \quad , \quad \frac{5}{11} \quad , \quad \ldots$$

(a) Write an expression for the ***n*th term** of the sequence.

1 mark

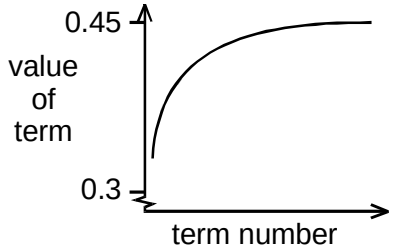
(b) The first five terms of the sequence are shown on the graph.

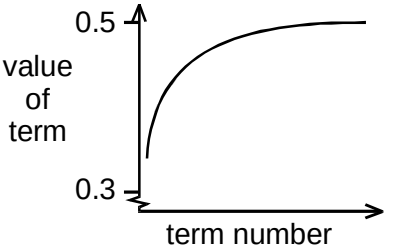


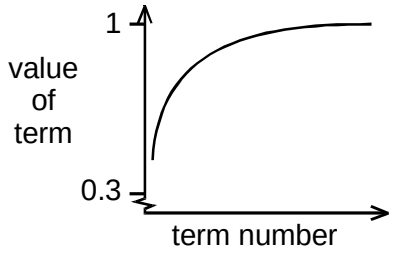
The sequence goes on and on for ever.

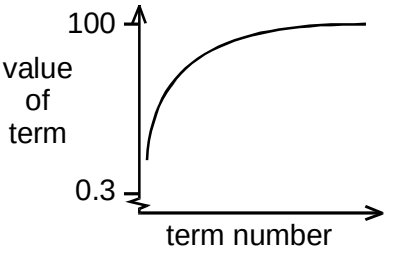
Which of these four graphs shows how the sequence continues?

Tick (√) your answer.









1 mark

(c) The *n*th term of a different sequence is $\frac{n}{n^2+1}$

The **first term** of the sequence is $\frac{1}{2}$

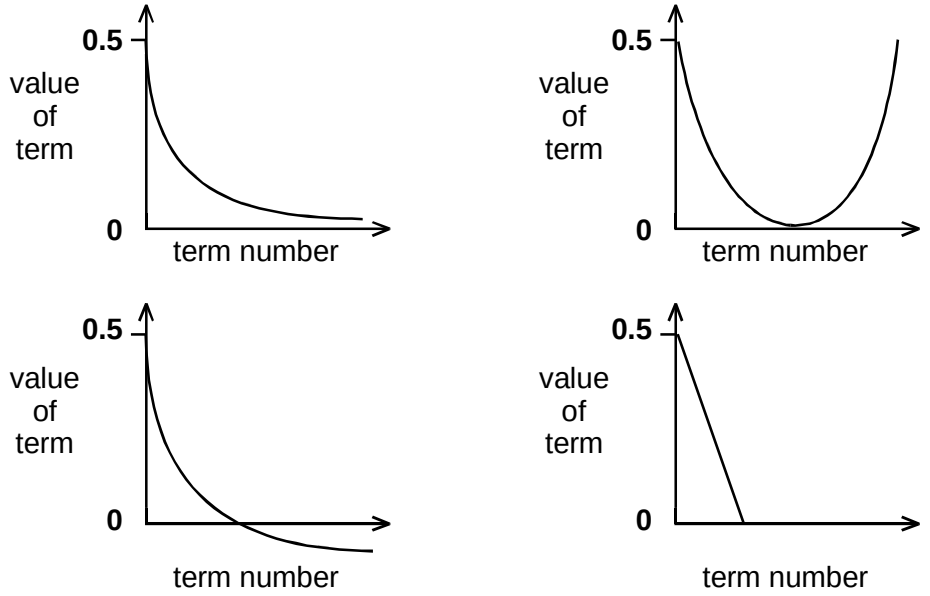
Write down the **next three** terms.

2 marks

(d) This new sequence also goes on and on for ever.

Which of these four graphs shows how the sequence continues?

Tick (√) your answer.



1 mark

Total 5 marks